Oxford Handbooks Online

Mesopotamian Mathematics

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Oxford Handbook of Science and Medicine in the Classical World
Edited by Paul T. Keyser and John Scarborough

Print Publication Date: Aug 2018 Subject: Classical Studies, Ancient Science and Medicine
Online Publication Date: Jul 2018 DOI: 10.1093/oxfordhb/9780199734146.013.61

Abstract and Keywords

The chapter explores “Mesopotamian mathematics,” which arose in the late fourth millennium bce, alongside a logographic script, both of which served in accounting. Writing, accounting, and calculation were in the hands of the manager-priests of the temples, who used the techniques to calculate and control land distribution to high officials, rations in kind to workers, and ingredients necessary for products such as beer. Mathematical texts include problems that seem practical but which would never occur in actual scribal work: their function was to display professional identity by exploiting a professional tool. The place-value system was created to simplify accurate calculations. Central to Old Babylonian mathematics were problems concerned with the properties of the sexagesimal system, as well as “algebraic” problems based on a set of four problems about rectangles with a given area, and some linear constraint. Such geometrical riddles have left traces in the pseudo-Heronian Geometrica collections and in medieval Islamic and Indian practical geometry and are likely to have inspired Euclid’s Elements II.

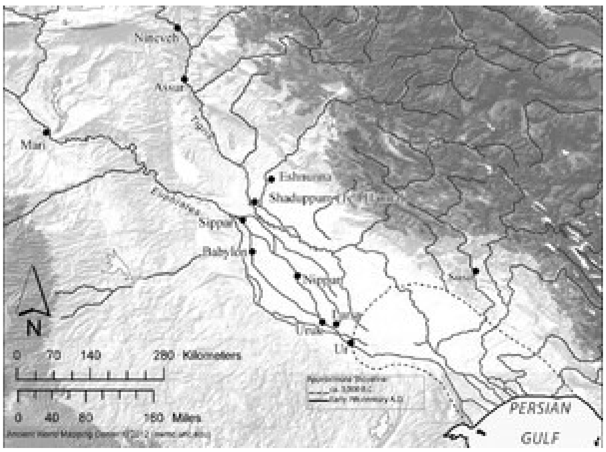
Keywords: accounting, algebra, mathematics, place-value, Pythagorean theorem, rectangles, riddles, sexagesimal

1. Social Base and Role, and Gross Development

THE complex we now call “Mesopotamian mathematics” was shaped in the late fourth millennium BCE (this, and all following dates use “middle chronology”) during the so-called proto-literate period, alongside a logographic script. The only function of both was to serve in accounting.

The context in which the complex emerged was the first formation of a bureaucratic state in southern Iraq around the city Uruk (map A1a.1). Writing, accounting, and calculation were the responsibility and privilege of the manager-priests of the temples, who used the techniques in the calculation and control of land distribution to high officials, of rations in kind to workers, and of necessary ingredients in the production for instance of beer (one of the acceptable ways to make barley consumable for humans) (Nissen, Damerow, and Englund 1993).

The mathematical techniques were based on notations for a counting sequence and for metrologies for lengths, for areas (geared to length measures), for capacity, and for time (namely, an administrative calendar used for the allocation of fodder and perhaps rations, decoupled from sun and moon). The number system had a base 60 (or rather, alternatingly 10 and 6, in the same sense as the Roman numeral system has alternating bases 5 and 2); the metrologies had step factors that were fairly convenient within the sexagesimal counting system (as the factor 12 between inch and foot) but were not themselves sexagesimal, which indicates that they had been created as normalizations of preexisting “natural” measures. We also have evidence of the computation of rectangular and near-rectangular areas from the sides—the former as inherent in the gearing (p. 12) of the metrologies, the latter by means of the “surveyors’ formula,” average length times average width.



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Map A1a.1 Sumerian and Babylonian sites.

Ancient World Mapping Center.

The evidence comes almost exclusively from discarded clay tablets that were used as filling material. This evidence is obviously incomplete yet complete enough to provide a good picture of mathematical practice. In particular it is clear that mathematics was taught in direct emulation of accounting routines: the only teaching texts we have are “model documents”—texts that lack an official’s seal and have nicer numbers than could be expected in real-life accounting but are otherwise indistinguishable from genuine accounting documents. Due to the pioneering efforts of Jöran Friberg (1978; 1979) and the computerized analysis of the complete corpus by Peter Damerow and Robert Englund (1987) we understand the numerical and metrological notations better than the linguistic aspect of the script.

The temple-centered Uruk state had an ill-documented breakdown in the early third millennium. During the ensuing “Early Dynastic” period, a polycentric system of city-states emerged, in which political power was taken over by a king (a military leader) and the temples became subordinate. For a couple of centuries, written evidence is almost completely lacking, but around 2600 BCE we find new accounting material from Ur (First Dynasty of Ur, famous for its Royal Tombs), and soon afterward much more from the city Shuruppak (shortly before that we also have the earliest royal inscription).

(p. 13) It is clear from Ur and Shuruppak that neither the writing tradition nor the mathematical techniques had been extinguished—their absence from the horizon during the intermediate centuries can only in part be due to rarefaction of the tradition. But Shuruppak (from whose epoch we also have the first literary texts, a proverb collection and a hymn [Biggs 1974]) tells us much more. First, *scribes* turn up as a particular profession (growing out of but also away from the class of temple managers), moreover with professional specializations (contracts written by one scribe refer to the presence of another one responsible for mensuration).

We also find an innovation in mathematics education: “supra-utilitarian” problems, that is, problems that seemingly deal with practice but would never turn up in real-life scribal work. They do not represent *theory*. From a modern perspective they look like “mathematics for fun,” in the style of riddles; at the time, their function was rather to manifest professional identity through testing and display of the scope of a professional tool (just as the writing of literary texts tests the other main tool); and it would be mistaken to impose a dichotomy pure/applied. One example asks for the distribution of the contents of a “granary”—known to consist of 40•60 “tuns” of 8•60 “liters” each—in rations of 7 “liters”; it exists in two copies, one of which is answered correctly, while the other calculation is either incomplete or wrong (Høyrup 1982). The merit of the problem is the division of a huge quantity by a factor that is at odds with the metrology (and which was never used in administrative practice). From Shuruppak we also have the first “table of squares”—*geometrical* squares, given in area metrology, corresponding to sides given in length measure.

We know Shuruppak (or rather its last year) so well because the city was destroyed in a military attack. The next informative phase is the “Old Akkadian period” (2335–2193), during which first southern Iraq, and then for a while a much more extensive area, was united into or controlled by a single regional state established by Sargon of Akkad (the so far unidentified city in central Iraq which gave its name to the “Akkadian” language, of which Babylonian is a dialect). Scribes and scribal culture lost nothing of their importance because of the change. Literature, invented in the preceding period as an expression of scribal identity, was adopted as royal propaganda bolstering the legitimacy of the new state. Supra-utilitarian mathematics could serve no similar purpose. However, the scale of administration and accounting grew (Foster 1982). Moreover, the epoch has left a number of supra-utilitarian school problems about area calculation—for instance, asking for one of the dimensions of a rectangle of which the area and the other dimension are known (Foster and Robson 2004). These problems are not “striking” or “funny,” and they may perhaps reflect autonomization of mathematics teaching from immediate practice rather than the formation of professional identity. In any case, they contain the rudiments of a specific vocabulary for problem solution (on which more below).

In spite of the incipient introduction of supra-utilitarian problems, Early Dynastic and Sargonic mathematics was first of all an accounting and mensurational technique, and thus centered on metrology. A new weight metrology was created from scratch, with all step factors equal to 60—except for the step between the "shekel" and the "barleycorn," which differed by a factor 3•60 (obviously because the barleycorn is (p. 14) a natural measure that could be normalized but not changed radically). The same trend of "sexagesimalization" can be seen in the upward and downward extension of existing metrologies. However, when a Sargonic "royal" capacity metrology was introduced—probably not meant to replace the local metrologies but to serve royal administrative purposes—the traditional structure was only modified to fit bureaucratic procedures, which were allowed to overrule the purely mathematical rationality of sexagesimalization.

The next centralization of power, under the Third Dynasty of Ur (2112–2004; or Ur III), overcame the contradiction between mathematical and bureaucratic rationality, and thereby did much more. The context was a centralized economy (established ca 2075, probably as part of a military reform); a large fraction of production was taken care of by workers' troops managed by overseer scribes—how large is disputed and probably undecidable because production outside state management would leave relatively few written traces. The scribes were responsible for the output of their crew, calculated according to fixed norms in units equivalent to 1/60 of a working day, or corresponding amounts of grain or silver—the debit being calculated from the number of workers (male, female, children) allotted to the overseer and borrowed from other crews, the credit from the yield and from the number of workers who were loaned to other scribes or who were ill, deceased, or in flight. The overseer's yearly deficit was accumulated, and if at his death his family could not cover it, its members and household slaves would be thrown into the labor troops as slaves—"that deficit is (therewith) resolved," as one accounting text states (Englund 1991, 268).

In its principles (Englund 1990, 13–90), the accounting system was thus not far from double-entry bookkeeping. Its use asked for a huge amount of calculations. As a simple illustration, we may think of the task of carrying the bricks of a certain type for a wall of given dimensions over a certain distance. Even if one knew how much a worker was supposed to carry a day, to find the corresponding number of working days would be prohibitively complicated if calculations were made in traditional non-sexagesimal metrologies.

The solution was the creation of a new numeral system: a floating-point place-value system with base 60, accompanied by "metrological tables" translating all units and relevant multiples into place-value multiples of a basic unit for each metrological system, and by tables of technical constants telling, for example, how many bricks of a given type a man could carry a unit distance in one day (say, *a*), and how many of them went into a unit volume (say, *b*). Then the dimensions of our wall would have to be expressed in place-value numbers and multiplied together, the product multiplied by *b* and next divided by *a*. The multiplications could be made by means of tables of multiples, and the division as a multiplication by the reciprocal of *a* (technical constants were always chosen so as to possess a finite reciprocal); both tables of multiples and of reciprocals were learned by heart. Obviously, this floating-point system was only used for intermediate calculations; final results were written down in the traditional notation, in which the order of magnitude was well-defined.

The basic idea of place-value sexagesimalization had been experimented on for centuries, but all texts preceding Ur III that try to make use of it contain errors (Powell (p. 15) 1976), showing that *the system* was not yet there. Without the complete system, including metrological tables and tables of multiples, reciprocals and technical constants, place-value computation was of no use. Addition and subtraction, indeed, had no need for place-value operations: they had been performed on some kind of reckoning board known as a “hand” at least since Shuruppak (Proust 2000; Høyrup 2002b).

The planned introduction of a complete technical system, mathematical or otherwise, has few parallels until recent centuries. The “place-value *system*” was, indeed, not only mathematical but also eminently social: it could only work if its users were thoroughly trained in an adequate teaching institution. Of this institution we know nothing except through its continuation in the ensuing “Old Babylonian” period—but it must have been there from the start; comparison with such systems as the Assyrian military machine or modern railway structures, also technical as well as social, is thus not off the mark.

The implementation of a place-value system within a single generation may be confronted with the millennium or more it took for the Chinese rod numerals to give rise through spontaneous development to a genuine place-value notation (Martzloff 2006, 185–188). The cost (which the creators of the system hardly considered a cost) appears to have been complete elimination of the culture of mathematical *problems* and of supra-utilitarian mathematics (Høyrup 2002c). As in proto-literate times, the only mathematics teaching texts we know from the period are model documents and arithmetical tables probably meant for training.

The Ur III Empire lost control of its peripheral conquests in 2025, and the core itself dissolved into smaller states in 2004. The initially leading successor (Isin) attempted to continue Ur III ideologically; to a gradually lessening extent, centralized management also persisted, but in the 18th century we see both increased weight of private management and an ideological impact of this change. The private person steps forward in a way that was unknown before—private letter writing, private seals, and even personal tutelary gods turn up. In scribal ideology (as known from the texts used to inculcate professional pride in the scribal school), an ideal of being *particularly human* appears. This “scribal humanism” emphasized supra-utilitarian abilities: ability to read and *speak* Sumerian—the dead prestige language of the scribal tradition—and familiarity with occult meanings of cuneiform signs. Mathematics used in surveying and accounting also enter the list, but the texts are not specific beyond that.

However, mathematical texts are more informative. Throughout the Old Babylonian epoch (2000–1600) the place-value system and all its appurtenant tables were trained together with the determination of simple areas and volumes; from the city Nippur the material suffices to reconstruct the complete syllabus (Robson 2002; Proust 2008). All the more strangely, that level of sophisticated supra-utilitarian mathematics that is commonly known as “(Old) Babylonian mathematics” is virtually absent from Nippur, where Sumerian literature was taught at the same time as simple multiplications (as also elsewhere); we must conclude that sophisticated supra-utilitarian mathematics was not part even of the full normal curriculum but was only practiced in specialized schools (the texts in question are in an indubitable school format).

(p. 16) The first evidence for the appearance of this kind of mathematics comes from 19th or very early 18th-century Ur in the south (Friberg 2000) and from contemporary Mari in the extreme northwestern periphery (Soubeyran 1984). The mathematics of the texts from Ur seem to descend primarily from the Ur III tradition, but with two conspicuous innovations: it sometimes serves for the creation of supra-utilitarian problems; moreover, these problems are sometimes structured in a rudimentary *problem format*, asking the question by means of the logogram en.nam, “what,” and stating that the result is “seen.” Already the Old Akkadian texts had “seen” results; but since they had asked the question by means of the possessive suffix .bi, “its [length, etc.],” absent from the Ur problems, direct transmission through the Ur III school is unlikely.

Mari had never been part of the Ur III Empire, but at some moment it must have borrowed the place-value system, superimposing it upon and amalgamating it with its own decimal counting system (as happened in much of the Syrian area, cf. Chambon 2011). Most of the mathematical texts that have been found are tables of multiples, inverses, and inverse squares. One, however, is quite different, namely a calculation (without problem format) of the famous “grain on a chess-board problem” (though ascending only to 30, which was usual until the invention of chess). It shows where Old Babylonian mathematics teachers could find inspiration for supra-utilitarian mathematics: namely from traditions of mathematical riddles carried by practitioners’ traditions (in the present case probably traveling merchants—Mari was on an important trade route).

The important step, however, seems to have been taken in Eshnunna, a state in mideastern Mesopotamia that *had* been subject to Ur III between 2075 and 2025, and which in general appears to have been the cultural center of central Mesopotamia around 1800 (before the rise of Babylonia).

The earliest mathematical Eshnunna text, from ca 1790, contains a problem about the subdivision of a right triangle (Baqir 1950a). It uses a format sufficiently close to that of Ur to suggest a connection and sufficiently different to exclude direct descent. More interesting is a rather large group of texts from ca 1775 (Baqir 1950b; 1951; 1962; al-Rawi and Roaf 1984; Gonçalves 2015; etc.). On one hand, it shows familiarity with almost all the main problem types that characterize mature Old Babylonian mathematics, in particular the so-called "algebra" (on which more below). On the other, it displays rather elaborate formats, homogeneous in subgroups but varying between these and unequally developed—implying that a canonical way to write problems was deliberately sought but agreement had not yet been reached. Inspiration for problems was taken in part from the Ur III computational tradition, in part from riddles circulating among nonscribal, probably Akkadian-speaking surveyors.

Eshnunna was conquered and destroyed by Hammurabi's Babylon in 1761 (and Mari in 1758). After that we know of no mathematical texts from the area, and its cultural role was taken over by Babylon; Hammurabi's famous law code could have been directly inspired by one produced in Eshnunna around 1790. Perhaps because the Old Babylonian strata of Babylon are covered by ruins from later epochs, we have no evidence that Hammurabi also brought mathematics or mathematics teachers to Babylon.

(p. 17) However, we do have sophisticated mathematical texts from the subsequent period. Almost all of them come from illegal diggings, and therefore only internal evidence allows us to determine their origin. However, a first division of the corpus into distinct groups was made by Albrecht Goetze (1945) on the basis of orthography; analysis of the terminology allows further refinement (Høyrup 2002a, 317–361).

A number of texts were produced in the former Sumerian cities Uruk and Larsa between the 1740s and the 1720s (by 1720 the south had seceded, and literate culture appears to have withered away). An interesting text from Larsa (AO 8862), roughly contemporary with a dated text from 1749, shows evidence of belonging to the earliest phase of a local development—especially a vacillating terminology. Other texts belonging to the same orthographic group confirm this.

Two distinct groups from Uruk seem mature. Both are highly standardized, reflecting a deliberate effort to develop a canonical format. However, almost everywhere a choice is possible, the choices of the two groups differ. The most likely explanation appears to be conflict or competition between two schools or teachers.

On one account, however, all of these texts, and all others from the south, agree: although an oblique reference shows the idiom to be familiar, they never state that a result is "seen," as done not only in the Old Akkadian texts but also in the problem texts from Ur and in many of the Eshnunna texts. The avoidance must be deliberate: what may have been brought to Babylon was reformulated to demarcate the southern developments from what (probably) was done in Babylon.

After 1720, perhaps before, many scholars from the south went north, and from the 17th century we have a number of sophisticated mathematical texts from northern Babylonia. Some of these may draw on traditions coming from the south, while a group from the town Sippar seems (according to terminology and closeness to practical surveying habits) to be local (and thus somehow related to the Eshnunna group).

In 1595, Babylonia fell first to a Hittite raid and next to Kassite warrior tribes. During the following centuries, traces of literate culture are rare, and the scholarly scribal tradition seems to have been kept alive in “scribal families.” These families conserved and systematized traditions concerned with language studies (not least Sumerian), omens, and medicine cum incantation; sophisticated mathematics appears to have been disregarded. Place-value computation and the appurtenant metrologies may have been remembered. In any case, the Assyrian king Assurbanipal not only collected the scholarship of the scribal families in his mid-7th-century library but also boasted of being able to multiply and find reciprocals. Less scholarly calculators probably also kept place-value calculation alive while developing new metrologies closer to the cares and ways of actual agriculture.

In the 5th century it seems that some of the scholar-scribes who were involved in the development of mathematical astronomy were also aware that sophisticated mathematics ought to be part of their interest. We have a few texts of “algebraic” character (in the same sense as in the Old Babylonian period), but their way to find Sumerian equivalents for Akkadian terms shows that these were reinventions; so, once more the inspiration appears to have been riddles belonging to Akkadian- (or, by now, Aramaic-) speaking surveyors.

(p. 18) Another couple of such texts were produced in the 3rd century within the same environment. Terminology, topics, and methods show that they do not descend from the 5th-century texts. Some of their characteristic problems turn up in Demotic Egypt at the same time; by then, Assyrian, Achaemenid, and Macedonian armies with their surveyors and tax collectors had been familiar visitors or masters of Egypt for half a millennium. All we may conclude is thus that the scholar-scribes this time borrowed from practitioners whose activity also made itself felt in Egypt.

In summary, accounting and mensurational mathematics had been a key ingredient in the formation of the first Sumerian state, and even when royal military power took over leadership in the Early Dynastic and Sargonic state, the scribal carriers of mathematical competence retained a high prestige—(cf. Visicato 2000). During Ur III, scribal competence, also in accounting mathematics, was something the king boasted of possessing—irrespective of the unpleasant social situation of those actually responsible for the accounts. Even during the mature Old Babylonian period, where accounting justice no longer served as legitimization of power, mathematical competence was still part of the same scribal curriculum as Sumerian literature, and thus shared the prestige of scribes.

In those phases where independent scribal professional identity existed, it also found expression in the devising of supra-utilitarian mathematics.

All this changed after the collapse of the Old Babylonian system. Subsequently, scribal scholarship and mathematical practice appear to have separated, being carried by distinct social groups. In the first millennium, when written evidence becomes abundant again, scholar-scribes became ritual experts and omen interpreters for the Assyrian rulers, whose correspondences with the scholars have survived. Material planning and accounting was certainly no less important for the Assyrian Empire, but the names of those who took care of such matters have not been preserved—they had no more cultural distinction than the practical calculators of Greco-Roman antiquity, and they wrote alphabetically on perishable material, not as the prestigious scholars in cuneiform on clay.

2 (Old) Babylonian Mathematics, and Afterlife

What is normally presented in histories of mathematics as Babylonian mathematics is the supra-utilitarian level of Old Babylonian mathematics, perhaps mixing in some Seleucid text without making any temporal distinction; this is also what is contrasted with, and sometimes connected to, Greek (theoretical) mathematics.

Actually, the scope of the higher level of Old Babylonian mathematics was wider than this—see for example the texts in Neugebauer 1935–1937; Neugebauer and Sachs 1945. (p. 19) Not everything is supra-utilitarian; we also find utilitarian problems about carrying bricks, the amount of dirt needed for a construction project, and so on—calculations an Ur III overseer scribe had been accustomed to perform. But supra-utilitarian problems were certainly central.

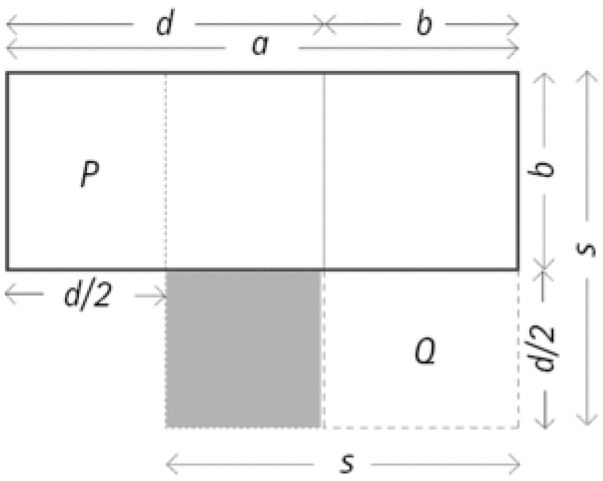

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Figure A1a.1 Babylonian rectangle problem, similar to Euclid, *Elements* II.6.

Drawing by author.

Some of these had to do with the properties of the sexagesimal system—for instance, an elegant method for finding reciprocals of difficult numbers. Many more, however, belong with the so-called “algebra”. (I here summarize some of the main results of Høyrup 2002a.)

It appears that the starting point for this discipline was a set of four problems about rectangles with a given area, for which was also known one of (1) the length, (2) the width, (3) the sum of length and width, or (4) the difference between these. (1) and (2) had already been dealt with in the Sargonic school; the trick by means of which (3) and (4) could be solved was probably discovered between 2200 and 1900 in an Akkadian-speaking lay surveyor’s environment, within which the problems are likely to have circulated as professional riddles.

The trick to solve (4) is shown in figure A1a.1: the area of the heavily drawn rectangle ⊏⊐(a,b) is known to be A, while the known difference between the sides is $d = a - b$. First, the difference is bisected, and the part P is moved to Q so as to form a gnomon together with the unmoved part of the rectangle. This gnomon still has area A. In its corner, the shaded square □($d/2$) is fitted in. This produces a completed square with area A + □($d/2$), whose side s is found. Joining $d/2$ to s gives the length a of the rectangle, removing it gives the width b.

The same trick can be used to find the side of a square □(c) if the sum □(c)+c = A of the area and the side is known—we just observe that □(c)+c = ⊏⊐(c+1,c). Even this seems to have existed as a surveyor's riddle.

The problem, as well as the procedure, is easily translated into equation algebra. In the square-plus-side version it becomes

$$c^2 + c = A \Rightarrow c^2 + c + \tfrac{1}{2}^2 = A + \tfrac{1}{4} \Rightarrow (c + \tfrac{1}{2})^2 = A + \tfrac{1}{4} \Rightarrow c = \sqrt{A + \tfrac{1}{4}} - \tfrac{1}{2}$$

(p. 20) (omiting negative numbers, which the Babylonians did not have). This is the primary reason that it has been customary to speak of "Babylonian algebra."

However, there may be better reasons (whether they are sufficient depends on taste and definitions). Indeed, the square and rectangle problems themselves are almost absent from the record. What we find are mostly complicated problems which, with transformations that correspond to linear equation manipulations, can be reduced to the simple problems, and others that do not deal at all with geometric entities but can be translated (as *we* may translate geometric problems into algebraic pure-number questions). One example (VAT 8520 #1, Neugebauer 1935–1937, 1.346) dealing with *igûm* and *igibûm*, a pair of numbers from the table of reciprocals, states in literal translation that

the 13th of the accumulation of *igûm* and *igibûm* to 6 I have repeated, from inside the *igûm* I have torn out, ½ it leaves

– in symbols, if a is the *igûm* and b is the *igibûm* (whence $a \bullet b = 1$)

$$a - {}^{6}\!/_{13}(a + b) = {}^{1}\!/_{2}.$$

This is transformed into a rectangle problem of type (3),

$$⊏⊐(7a, 6b) = 42,\ 7a - 6b = 6{}^{1}\!/_{2},$$

from which a and b are easily found. In other problems, prices, areas or volumes are represented by linear magnitudes.

When H. S. Schuster and Otto Neugebauer discovered this “algebra” around 1930, they took the geometric terminology to be a purely arithmetical imagery (as with us a “square number”), and Neugebauer (1936, 250) thought the so-called geometric algebra of *Elements* II to be a translation of a Babylonian arithmetical technique into geometry, undertaken in order to save its results from the philosophical threat or “foundation crisis” assumed to have resulted from the discovery of irrationality. Once it is realized that already the Babylonian technique was based on geometry, Babylonian inspiration seems even more plausible, despite Arpád Szabó (1969, 455–456)—as a matter of fact, the diagram in figure A1a.1 only differs from that of *Elements* II.6 by the absence of a diagonal by means of which Euclid performs the construction instead of just “moving around” a rectangle. (The diagram of rectangle problem (3) is equally close to that of *Elements* II.5.)

However, some difficulties remain. First, could the Greeks have known the Babylonian technique? This was doubted by Szabó, but the difficulty is worse than he knew, since what Neugebauer had spoken about was an *Old Babylonian* technique that had disappeared a millennium before Thales’s times. (The Seleucid texts make use of diagrams that do *not* correspond to *Elements* II, and the 5th-century problems, formulated in discordant area- and length-metrologies, are equally irrelevant.)

Second, *Elements* II solves no problems, at most its theorems can be claimed to correspond to algebraic identities, in which Babylonian texts (Old or Late) show no interest; this was also seen by Szabó. Third, we find nothing in Greek mathematics that (p. 21) corresponds to the fully developed Old Babylonian discipline, only counterparts (transformed into “identities”) of the original riddles.

We know, however, that the riddle tradition was still alive in the Islamic Middle Ages, and even reached medieval India (Høyrup 2001). It has also left traces in the pseudo–Heronian *Geometrica* collections. It is more than plausible that archaic Greeks encountered it in the same Aramaic-speaking region that gave them their alphabet. As the alphabet, it was probably not only adopted but also adapted; in any case, the earliest certain Greek evidence we have for it is precisely *Elements* II—the fragment of Hippocrates of Chios constitutes a plausible but indirect and not very informative trace, and oblique references in the Platonic corpus (Høyrup 1990) are no more certain. What we find here is not a technique but, so to speak, a *critique* of this technique, putting things on a metatheoretically firm footing and thus determining its possibilities and limits (*Möglichkeit und Grenzen*, in Kant’s words). This had been the aim of the Old Babylonian teachers to a very restricted extent only—in order to serve the professional identity of scribes, what they did had to remain supra-utilitarian: that is, to look relevant to the task of the calculating scribe, which had always been to *find the right numerical answer*. The limited amount of critique we find appears to be connected to pedagogical concerns—for instance, in explaining the method of rectangle problem (3) as above, to point out that a and b are found by moving *the same piece* $d/2$ back to its original position, for which reason it has to be removed before it can be joined. Early texts, indeed, do as above, and add before they subtract (for the Babylonians, as for us, this was the normal order); mature texts respect the concreteness of the procedure, doing it the other (concretely meaningful) way around.

On the elementary level, it is no wonder that the Greeks borrowed part of the metrology of their Phoenician trading partners, some of which was again an adaptation of Mesopotamian metrology. The sexagesimal place-value system was borrowed (along with many planetary parameters and information about observations) and used in Greek mathematical astronomy, though only for the fractional part of numbers (whence our "minutes," "seconds," etc.). During the Middle Ages and the Renaissance, it inspired several attempts to implement decimal fractions—ultimately successful in Simon Stevin's *De thiende* from 1585.

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